

Estimating Trajectory Correction Requirements for Multiple Outer Planet Missions

L. D. FRIEDMAN,* T. W. HAMILTON,† AND R. H. STANTON‡

Jet Propulsion Laboratory, Pasadena, Calif.

The estimation of trajectory correction (ΔV) requirements for multiple outer planet flyby missions is a unique and challenging problem for the systems analyst. This paper presents a general approach to solving this problem when the navigation system uses onboard optical measurements made during approach to each target planet to complement the ground-based radio measurements. The accuracy and reliability of the onboard measurement system plays a critical role in sizing the ΔV capability required. An illustration of the combined use of radio and optical measurements is provided for the particular case of a Jupiter-Uranus-Neptune (JUN) mission. Use of the statistical technique developed for computing the ΔV margin required to account for uncertainties in subsystem performance, permits ΔV savings of 100–120 m/sec over “worst case” designs. This represents weight savings of about 50% of the science payload. For the example case ΔV requirements are estimated for two candidate optical systems and the radio alone case. The use of onboard measurements allows a ΔV savings of approximately 140 m/sec.

Nomenclature

\mathbf{b}'	= unit vector normal to $\mathbf{v}_{\infty i'}$ in plane of motion
b'	= semiminor axis of spacecraft's hyperbolic trajectory relative to planetary satellite
F	= required fuel load
F_{NOM}	= nominal estimate of required fuel load
F_{WC}	= worst case estimate of required fuel load
$G()$	= probability of having sufficient fuel to meet navigation success criterion
P_k	= probability of meeting nominal measurement performance goals
P_O	= probability of optical system working in predicted mode
P_N	= a specified navigation success probability
\bar{P}_N	= average value of P_N , realized over an ensemble of development outcomes
$\mathbf{v}_{\infty i'}$	= incoming asymptotic velocity of spacecraft relative to planetary satellite
δ	= declination of spacecraft
$\delta\mu'/\mu'$	= fractional mass error of satellite
ΔF	= “design margin” in the required fuel load
ΔV	= generic symbol for speed change required for trajectory correction due to navigation
ΔV	= corrective velocity required for a particular maneuver
ΔV_{∞}	= required ΔV due to satellite position and mass errors applied on outgoing planetocentric asymptote
Φ	= a propagation matrix relating planetocentric spacecraft velocity components at the satellite passage to those on the asymptote
ν'	= bending angle of spacecraft hyperbola relative to satellite
σ_{IN}	= predicted standard deviations of one-station ($i = 1$), two-station ($i = 2$), ephemeris ($i = 3$) and optical ($i = 4$) measurements

Submitted March 16, 1972; revision received July 19, 1972. This research was sponsored by NASA under Contract NAS7-100. We would like to thank C. Lefkowitz for his aid in obtaining the ΔV estimates.

Index categories: Spacecraft Navigation, Guidance and Flight Path Control Systems; Unmanned Lunar and Interplanetary Systems; Navigation, Control and Guidance Theory.

* Member Technical Staff, Mission Analysis Division. Member AIAA.

† Manager, JPL Navigation Program, Mission Analysis Division. Associate Fellow AIAA.

‡ Member Technical Staff, Guidance and Control Division. Associate Member AIAA.

Background

FOR the past few years efforts have been made to formulate mission designs for multiple outer planets missions. Such designs must accommodate weight, cost and risk constraints while maximizing the scientific return potential.

A major element of mission design is the synthesis of the navigation system. Weight and cost restraints for the outer planet spacecraft require that the navigation system be optimally designed to conserve trajectory correction fuel (ΔV capability). The selected navigation system configuration must balance the weight, cost and risk characteristics of its subsystems. The associated trajectory correction requirement must, in some measure, reflect statistical estimates of subsystem performances.

The proposed navigation measurement subsystem for use on the multiouter planet missions has been described previously.¹⁻⁵ It is the measurement subsystem which dictates the philosophy of the navigation system design. To augment the radio tracking data, onboard TV pictures of natural satellites against the star background will be used to infer the spacecraft's location in planetocentric coordinates.² The radio part of the navigation system will use improved planetary ephemerides and multireceiver (quasi-VLBI) data.^{4,5} This combined with the onboard information forms the navigation system.

Navigation system design has an immediate effect on the over-all mission design parameters of weight, cost, reliability and scientific return. The tradeoffs involved in synthesizing the system are complex. Since the mission design critically depends on these tradeoffs, they must be arrived at early in the design—before the detailed subsystem analyses can be done. This paper describes the analytical techniques which allow system tradeoffs to be performed despite lack of firm performance data for the subsystems. The sensitivity of the resulting trajectory correction (ΔV) requirements to these performance variations can be directly measured.

Navigation System Description

Earth-based radio tracking and ground command have been the basis for navigation to the nearby planets. As we extend our reach to the outer planets we are fortunate in that new radio data types which overcome the geometric singularities of conventional Doppler and ranging are becoming practical.

By simultaneously receiving Doppler and ranging at two stations during their overlap period we form an interferometer with an intercontinental baseline. The interferometer's angular accuracy is best when the spacecraft is in the Earth's equatorial plane (zero declination) which provides a vital complement to the Doppler data, whose declination accuracy degrades as cotangent of declination. The practical limit of the radio navigation system is the accuracy of our knowledge of the ephemeris of the target. Our knowledge of the target's ephemeris is seriously degraded by the present lack of radar observations of the outer planets and the reduced accuracy of optical observations due to 10–25 times greater geocentric distances involved.

The improved radio navigation system, teamed with on-board target direction information such as that obtained from the science vidicon on Mariner 9,⁶ provides the necessary capability to navigate the demanding multiple encounter outer planet missions under consideration. The accuracy of the target flybys and the total correction ΔV budget required to assure a given probability of being able to navigate to the final planet are the critical performance parameters. The Jupiter-Uranus-Neptune, 1979 (JUN' 79) mission is a good example of a ΔV budget limited situation. Because of the large payload fraction which must be allocated for navigation fuel we have directed considerable attention to the development of valid statistical methods to achieve realistic ΔV budget estimates for a relatively complex system in the face of uncertainties in ultimate subsystem performance.

The ΔV allocation to provide a specified navigation success probability P_N , assuming that we are certain of achieving nominally specified performance is straightforward to compute. Navigation is successful if enough fuel is left to target to the final planet. The probability of navigation success, in this case, is

$$P_N = P_O G(F; \sigma_{1N}, \sigma_{2N}, \sigma_{3N}, \sigma_{4N}) + (1 - P_O) G(F; \sigma_{1N}, \sigma_{2N}, \sigma_{3N}) \quad (1)$$

The ΔV required is obtained by solving for F . The sensitivity of the solution is illustrated in Figs. 2–4, for discussed later, typical values of P_N , P_O and σ_{1N} .

ΔV Requirement Modified for Performance Uncertainties (Margin)

If the actual flight performance could be assured to be described by the nominal values, the ΔV sizing job would be complete with the solution of Eq. (1). The possible range of the critical measurement performance parameters is shown in Table 1. The wide ranges come from uncertainty in the extent of accuracy improvements and from different candidate optical sensor designs.

Assuming the “worst-case” for all the important parameters cannot be afforded because little weight would be left for payload or for spacecraft redundancy. A more realistic assumption is to assume that nominal performance goals are met with probability P_k (e.g., 80%), and worst-case values are obtained

with probability $1 - P_k$ (e.g., 20%), over a statistical ensemble of realizations of development program results. It is convenient to write the new correction requirement as $F + \Delta F$, where ΔF is the “design margin” needed to cover performance uncertainties. We require that the correction capability be sufficient such that the average value of P_N , \bar{P}_N , realized over an ensemble of development outcomes equals P_N .

In performing the ensemble average we assume that the outcomes are independent for the four measurements. For the simple assumptions used, 16 discrete outcomes may be easily enumerated. From performing the calculations we have found a good approximation for ΔF

$$\Delta F \approx (1 - P_k)(F_{WC} - F_{NOM}) \quad (2)$$

where F_{WC} is the capability required if we solve Eq. (1) using all worst-case performances and F_{NOM} is the solution with nominal performances. When we assume $P_k = 0.8$ the savings are 0.8 ($F_{WC} - F_{NOM}$), resulting in ΔV reductions of 100–120 m/sec on the JUN mission. Payload thus made available for non-navigation purposes is over 50% of the designed science payload.

In the example just described we have used a very simple model for the statistical distribution of performance realizations. Any other specified distribution of performance realizations can be dealt with using the same approach; viz. that the ensemble average, \bar{P}_N , equals the required probability of success.

ΔV Estimation

Error Source Assumptions

The relevant parameters in the navigation system analysis are: 1) mission trajectory, 2) planetary ephemeris values, 3) single-station tracking—equivalent station location errors, 4) dual-station tracking—difference in arrival time between stations, 5) required probability of navigation success, 6) optical system accuracy, 7) optical system—camera reliability, 8) ΔV required for correcting satellite mass uncertainties, 9) ΔV required for planetary quarantine biasing, 10) cruise orbit determination capability, 11) maneuver execution errors, and 12) control of time of arrival requirement.

Nominal values are chosen as the expected values of performance. In the following we present some of the sensitivities of the ΔV for these parameters. These sensitivities will be used in the ΔV margin calculation. The ΔV values are partial requirements only and should be used as indicators of sensitivity only. The JUN 79 mission is used for illustrative purposes.

The sensitivity of ΔV to trajectory selection on JUN 79 is depicted in Fig. 1, where partial ΔV requirements are plotted

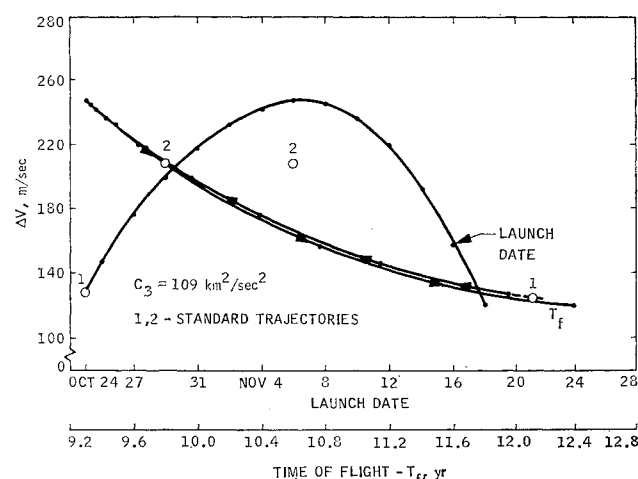


Fig. 1 Sensitivity of ΔV to trajectory.

Table 1 Measurement accuracy ranges

Measurement type	Accuracy measure	One-sigma error range
One-station tracking	Spin-axis distance	0.6–1.2 m
Two-station tracking	or Declination	(0.12–0.24)/tan δ μ rad
	Arrival time	1–2 m
Ephemeris error	or Declination	(0.2–0.4)/cos δ μ rad
	Declination or rt. ascension	0.25–0.5 μ rad
Optical accuracy	Angular error	30–80 μ rad

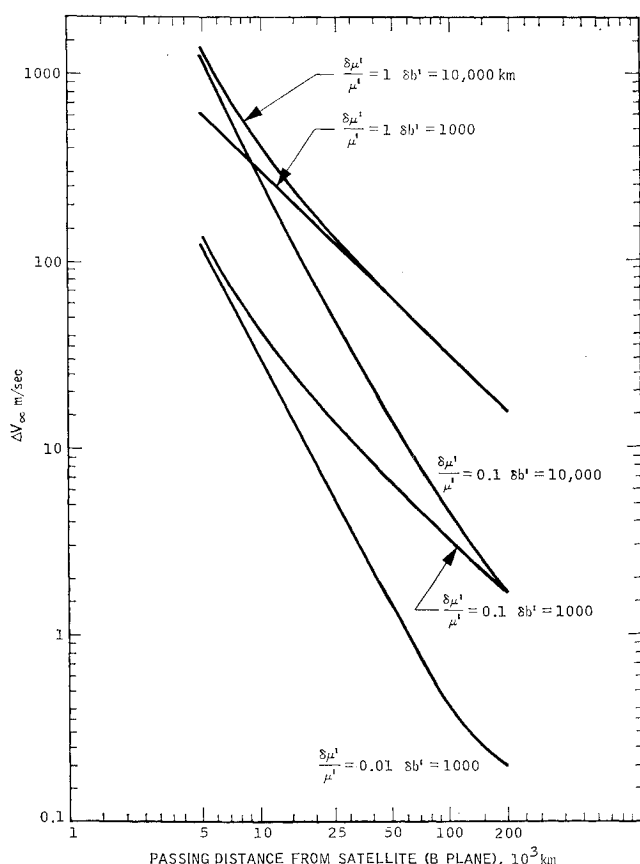


Fig. 5 ΔV for satellite flyby on JSP 77 at Titan.

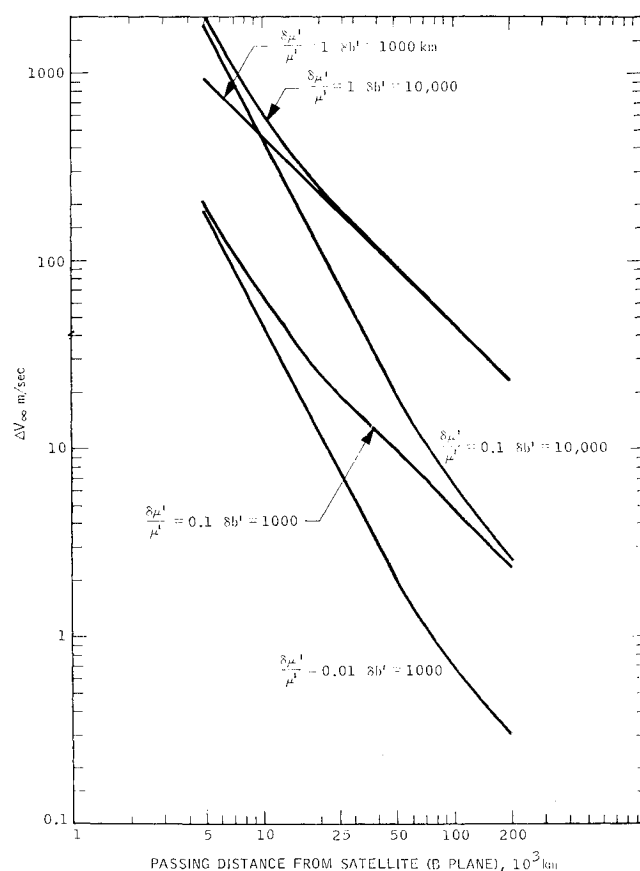


Fig. 6 ΔV for satellite flyby on JUN 79 at Europa.

in these studies, is the value of the launch vehicle covariance matrix of injection errors. Using a given covariance matrix, the amount of bias removal ΔV required (on the first mid-course maneuver) was 18 m/sec. The Jupiter flyby must also be biased to avoid Saturn contamination. Using a conservative value of the probability of performing the post-Jupiter maneuver bias removal ΔV of 5 m/sec on that maneuver for JUN 79 was found to be required.

The Onboard Approach Measurement System

The performance of the total navigation system and the related ΔV requirements are strongly tied to the characteristics of the onboard optical measurement system. As seen in Fig. 3, if either the accuracy or reliability of that system is lacking, the resulting fuel loading penalty can be severe. For this reason, some of the factors affecting both the performance of the onboard measurement instrument and its reliability will be discussed.

In the basic observation mode, natural satellites of each target planet will be viewed against available star backgrounds. In some instances, a number of brighter stars are available, but for the general case it is necessary to detect increasingly fainter stars as the field of view (FOV) is decreased. Thus, while the detection threshold can be magnitude 7.5 for a square field 40 mrad on a side, a 12 mrad FOV requires detection of stars an order of magnitude fainter (10th magnitude). Moreover, in some cases the satellites to be tracked are as bright as minus 5th magnitude, 10^4 – 10^6 times brighter than the background stars.² When these values are compared to a typical vidicon dynamic range of 10^2 – 10^3 , it is evident that considerable overexposure of satellite images may result in frames exposed for adequate star detection. The resulting image spread and distortion may considerably degrade satellite position measurements.

Why not avoid this problem by making the onboard measurements with sensors other than vidicons? Instrument designs based on image dissectors or other scanning sensors can reduce or eliminate dynamic range problems, enabling highly accurate satellite/star measurements to be made even for very bright targets. Vidicons are the prime candidates, at least for missions launched in the 70's, since they can serve the dual purpose of providing scientific imaging data when not being used for navigation. Building separate instruments for scientific imaging and onboard trajectory measurements is justified only where a single instrument cannot satisfy both needs. Thus, if problems such as star detection and bright image blooming can be overcome with a vidicon design, this type of instrument will undoubtedly be used for the required satellite/star measurements.

Typically, planetary missions are flown with two vidicon imaging systems onboard: a wide angle (WA) camera to provide mapping coverage and a narrow angle (NA) camera to provide high resolution photography of selected areas. In the following discussion we assume that the outer planet spacecraft carries a system with camera fields of view of 100 mr (5.7°) and 20 mr (1.15°), respectively.

Measurement Accuracy

In addition to the blooming effects mentioned previously, other instrument problems lead to similar degradations in measurement accuracy. Electromagnetic field irregularities can produce frame-to-frame variations in image geometry. These can be reduced if adequate numbers of fiducial marks are visible against the black background. Image shifts resulting from electron beam pulling effects are often significant and must be extensively calibrated. Similar shifts can be introduced by the onboard video data compression system, particularly where the large intensity changes associated with star images or satellite limbs occur.

All of these effects can be calibrated and sizably reduced by Earth-based computer processing of the video data. Therefore, two questions must be answered in order to predict the

measurement accuracy of the approach measurement subsystem. First, how large are the various errors introduced by the camera? Second, how accurately can these errors be removed by ground processing of the data (remembering, of course, that such processing must be completed during real-time operations)?

Answers to both of these questions are only tentative at this time. The wide- and narrow-angle cameras differ considerably in imaging potential. The narrow-angle camera, depending on the sensor, optical aperture and exposure time used, may have difficulty detecting adequate numbers of stars, particularly for "star poor" backgrounds. However, the small field of view for this camera results in a greater angular resolution than can be achieved with the wide-angle camera. The wide-angle camera, while having less resolution, generally suffers less from dynamic range effects simply because its FOV will probably include much brighter stars. Other factors, such as distortions introduced by intensifying stages and whether or not white fiducial marks are available, all affect the ultimate accuracy that can be achieved. For these reasons, the expected measurement accuracy cannot be scaled with camera resolution (or FOV). Each instrument design must be considered separately in any accuracy analysis.

Significant progress has been made with the successful demonstration of the use of onboard optical measurements as part of the complete navigation system on Mariner 9.^{6,10} Although problems such as satellite image blooming were less significant at Mars than expected at the outer planets (due to the small size of the Martian moons), the in-flight achievement of accuracies on the order of one camera resolution element is definitely encouraging.

The optical observation accuracy assumptions used in the calculations of this paper are based on a preliminary analysis of the instrument and software considerations mentioned above. The large number of unknowns have dictated selection of the following conservative estimates. For the wide-angle camera it is assumed that the satellite position can be measured with respect to the stars to $250 \mu\text{rad}$ (3σ), which corresponds to 2-3 resolution elements. Similarly, $100 \mu\text{rad}$ (~ 5 elements) has been adopted as the error for the narrow angle camera due to the greater difficulty of achieving resolution limited accuracy with this system.

Measurement Reliability

Questions relating to the reliability of obtaining adequate optical navigation data at a given encounter are even more difficult to answer. However, some estimate of the conditional probability that the onboard cameras will fail while the rest of the spacecraft is operational must be made. Any failure that negates the scientific potential of the mission (e.g., radio failure) should clearly be ignored in the reliability analysis, since then the mission is effectively lost, regardless of the navigation performance.

Since there is more than one onboard camera capable of supplying useful navigation data, two types of failures must be considered. The first, common mode failures, have the property of degrading or eliminating data from both cameras simultaneously. These failures may arise either from the cameras themselves (e.g., damage caused by radiation at Jupiter) or from elements that are common to both cameras (e.g., data processing circuitry failures). Less serious are single camera failures, or all malfunctions where the performance of one camera is impaired independently of the other. Most wearout failures (cathode burnout, etc.) are of this second type.

When the appropriate failure probabilities are known (one for each camera and one common mode) these data can be combined with the measurement accuracy of each camera to yield estimates of the total capability of the onboard system. This was done and the ΔV estimates given in the following are obtained using values of 0.90 for the reliability of each camera

and 0.95 for the reliability of all common elements. These numbers, based on estimated parts counts and available component failure rates, are calculated for the Uranus encounter of a typical JUN mission since navigation errors at this planet size the ΔV requirements. Both cameras are assumed to have operated approximately 2000 hr. prior to launch and 2000 hr. during Jupiter encounter. In addition, a failure rate during dormant periods of $\frac{1}{10}$ of the operational failure rate is assumed for all cruise periods. Because of the uncertain nature of these failure probabilities, they can only be regarded as nominal values rather than worst-case estimates. Estimates for the latter of 0.8 per camera and 0.9 for common elements were included in the margin calculation. Calculations of the ΔV requirements for any mission thus include allowances for the probability that the optical data system will fail and the added margin to account for the fact that current estimates of reliability and accuracy are uncertain.

Analytical Calculations

To provide the calculations for the quick estimate of ΔV requirements for various subsystem configurations and trade-offs, we employed several simplifying assumptions. The rms ΔV requirement of a given maneuver was assumed to be the 1σ B-plane (magnitude) error being corrected, divided by the time to encounter from maneuver. For the initial preplanet maneuver (made during the approach), this B-plane error was assumed to be solely due to execution errors from the previous maneuver. For the final preplanet and for the postplanet maneuver, the B-plane error was assumed to be the result only of the orbit determination covariance.

The final encounter (E) orbit determination accuracy, assumed obtained at E-4 days, was a key parameter in the study. We assumed that single-station and multistation tracking operate in parallel. The resulting radio-only system accuracy is the root-mean-square of the ephemeris and radio angular accuracies. The optical system is assumed to be performing near its limiting distortion accuracy with random error in the measurements assumed to be filtered.

ΔV margin was calculated assuming worst-case values of measurement accuracies were twice the nominal and of reliability as described previously.

The rms value of each maneuver (along with the planetary quarantine and satellite flyby ΔV , if appropriate) is calculated and the mean and variance for the total mission ΔV is calculated assuming a certain probability distribution for each maneuver. (The individual maneuvers were assumed to be half-normal distributed. This has been found to be accurate assumption in the past, and several Monte Carlo checks with the present cases have allowed us to infer its validity here.) The statistics of the total ΔV are calculated for each mode of the combined radio-optical system operation: radio plus nominal optical, radio plus backup optical, and radio alone.

By assuming a probability distribution for the total ΔV (possibly different from those for the individual ΔV 's) we can then calculate the ΔV for any desired probability of success by Eq. (1). To this value we add the margin as previously described.

For the assumed probability distributions of the total ΔV we experimented with generalized (to 2-parameter) Rayleigh and half-normal, Weibull and gamma distributions. On the JUN 79 mission, one maneuver is dominant over all the others, i.e., the post-Uranus ΔV , and thus the resultant distribution was found to be well approximated by the generalized half-normal distribution.

ΔV Requirement Results for the JUN Mission

For the assumptions given above and for two candidate optical navigation measurement subsystems, we have calculated ΔV requirements for the example JUN 79 mission as a function of P_N , the probability of navigating successfully to the last planet. Total ΔV results for the two systems are shown

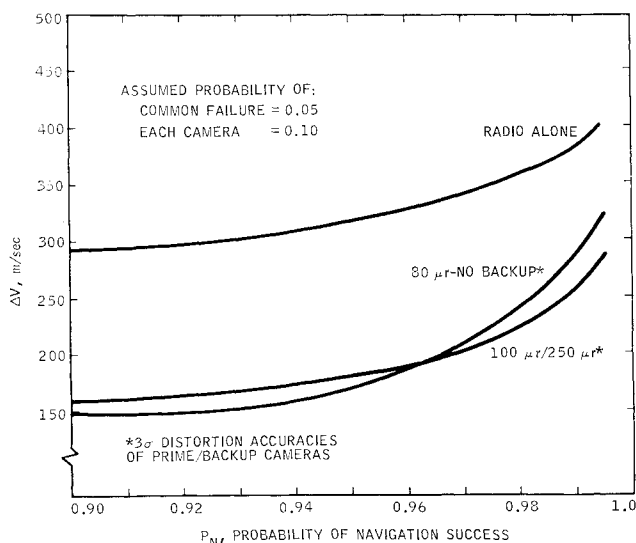


Fig. 7 ΔV requirements for JUN 79.

in Fig. 7. Also shown are results for the radio-alone navigation. For the baseline optical system (3σ accuracies of $100 \mu r$ prime, $250 \mu r$ backup) and $P_N = 0.98$ the requirement on the mission is 227 m/sec. The radio-only requirement would be 365 m/sec.

In order to scale the ΔV requirements to the error sources and to illustrate the magnitude of each maneuver we have presented in Table 2 a breakdown of the ΔV calculation for the baseline optical system, including margin.

From this preliminary parametric examination of ΔV requirements we have drawn the following conclusions: 1) Optical approach navigation is necessary for ΔV minimization on the example mission. In addition, its presence will enhance aiming accuracies and make possible closer satellite encounters. The performance of the optical system is especially crucial at Uranus on JUN 79. 2) Ephemeris knowledge of the outer planets and their satellites is important. Reduction of present uncertainties is crucial to minimizing ΔV and the payoff from the amount of reduction is significant (see

Fig. 4). 3) Considerable system level tradeoff and analysis study is necessary to evaluate candidate navigation systems and their requirements. Weight and cost ramifications must be carried through the entire spacecraft and mission design before final conclusions are possible. 4) The post-Uranus ΔV is the single largest factor in sizing ΔV requirements for the example mission. Nearly linear payoff or penalty in ΔV is obtained with improvements or degradation in pre-Uranus aiming accuracies.

Summary and Conclusions

We have described the basic navigation system for outer planet missions. Optical measurements taken near the target provide the necessary information to reduce the errors due to uncertainties in the target ephemeris. The radio and ephemeris information provide a valuable functional backup of lesser accuracy. We have described the analytical approach to obtain realistic ΔV budget estimates using the criteria of navigation success probability. The technique takes into account the conditional failure probability of the onboard measurements and our present limitations in predicting in-flight subsystem performance parameters. The effect of these unreliabilities and uncertainties is summarized in the form of a ΔV margin required to adequately cover them. Thus these factors can be quantitatively entered into the mission design process and realistic tradeoffs performed. Once a design is selected subsystem performance predictions can be monitored and updated, thereby allowing continuing assessment of the weight which must be allocated to trajectory corrections.

To illustrate the techniques developed we have used a JUN 79 mission as an example. By including all other considerations which make up the total ΔV budget for this mission we hope to give a better understanding of the entire ΔV allocation process and the role of the newly developed techniques in facilitating a realistic mission design. This more rationally based approach provides a basis to steer a middle course between over-conservative design assumptions which penalize the mission return and a disregard of the uncertainties of predicting the outcome of the subsystem design process.

References

- Ball, J. and Duxbury, T., "Navigating the Grand Tours," *Astrodynamics and Aeronautics*, Vol. 8, No. 9, Sept. 1970, p. 73.
- Stanton, R., "Development of an Approach Guidance Instrument for the Outer Planets," AAS Preprint 71-119, June 1971, Seattle, Wash.
- Hendricks, T. and Boain, R., "Navigation Analysis of a Jupiter-Uranus-Neptune 1979 Mission," AAS Preprint 71-117, June 1971, Seattle, Wash.
- Curkendall, D. and McDanell, J., "Investigation of the Effects of Major Earth-Based Navigation Error Sources for an Outer Planet Mission," AAS Preprint 71-118, June 1971, Seattle, Wash.
- Ondrasik, V. and Rourke, K., "Applications of Quasi-VLBI Tracking Data Types to the Zero Declination and Process Noise Problems," AAS Preprint 71-399, Aug. 1971, Fort Lauderdale, Fla.
- Duxbury, T. and Acton, C., "On-Board Optical Navigation Data from Mariner '71," *Proceedings of the National Space Meeting of ION*, Orlando, Fla., March 1972.
- Penzo, P., "The Outer Planets—What Should the Missions Be," AAS Preprint 71-137, June 1971, Seattle, Wash.
- Penzo, P., "Satellite Encounter Opportunities of the Grand Tour Missions," AAS Preprint 71-359, Aug. 1971, Fort Lauderdale, Fla.
- Gonzalez, C. and Stavro, C., "Flight Paths and Mission Strategies to Satisfy Outer Planet Quarantine Constraints," AAS Preprint 71-319, Aug. 1971, Fort Lauderdale, Fla.
- Acton, C., "Processing On-Board Optical Data for Planetary Approach Navigation," AIAA Paper 72-53, San Diego, Calif., 1972.

Table 2 Individual RMS maneuver values for baseline system and nominal (assumed) performance^a for the JUN 79 mission

Maneuver no.	Function	ΔV m/sec
1	Correct injection errors—RMS	18
1	Remove injection planetary quarantine bias	18
2	Correct execution errors—RMS	5
3	Correct orbit determination errors—RMS	3
4	Postplanet correct orbit determination errors—RMS	2
4	Correct satellite mass error	10
4	Remove pre-encounter planetary quarantine bias	5
5	Correct execution errors—RMS	7
6	Correct orbit determination errors—RMS	12
7	Postplanet correct orbit determination errors—RMS	34
8	Correct execution errors—RMS	6
9	Correct orbit determination errors—RMS	5
ΔV Margin		25
Time of flight control		3
ΔV mission requirement ($P_N = 0.98$)		227
ΔV mission requirement ($P_N = 0.99$)		251

^a For individual maneuvers, RMS values are calculated assuming $100 \mu r$ optical system performance, except at Jupiter.